UDC 539.18 – 047.58
DOI https://doi.org/10.32782/pet-2024-2-10

Petro TROKHIMCHUCK
PhD in Physics and Mathematics, Associate Professor, Associate Professor at the Department of A.V. Svidzynskiy's Theoretical and Computer Physics, Lesya Ukrainka Volyn National University, 13 Volya Ave., Lutsk, Volyn region, Ukraine, 43025
ORCID ID: *https://orcid.org/0000-0003-2737-0506*
SCOPUS-AUTHOR ID: *8383601100*

To cite this article: Trokhimchuck, P. (2024). Problemy modeliuvannia krytychnykh termoiadernykh protsesiv [Problems of modeling a critical thermonuclear processes]. *Physics and Educational Technology*, 2, 71–78, doi: https://doi.org/10.32782/pet-2024-2-10

PROBLEMS OF MODELING A CRITICAL THERMONUCLEAR PROCESSES

A brief analysis of the problem of modeling critical thermonuclear processes is presented. Attention was focused on two types of processes. First is determined by the generation of thermonuclear reactions in stationary regime. This problem is main for the creation thermonuclear reaction and has Earth value. Second is lifetime of stationary phase. This problem is main for lifetime of stars and have Universe value. The first refers to the problem of the threshold for the occurrence of thermonuclear reactions. Here, Lawson's criterion is analyzed and its significance in the problem of thermonuclear reactor construction is shown. Deuterium-deuteriun and deuterium-tritium reactions are analysed. Various mechanisms of modeling the generation and realization of these reactions, including magnetic fields, are discussed. The well-founded concepts of muon catalysis and its role in the generation of thermonuclear reactions are also given. The issue of the influence of the shape and symmetry of deuterium and tritium nuclei on the threshold for the generation of thermonuclear reactions and its contribution to the Lawson criterion is analyzed. The second part refers to astrophysics. The Schönberg-Chandrasekhar criterion is formulated. The Schönberg -Chandrasekhar theory of the residence time on the main sequence of the Hertzsprung-Ressel diagram, which is general for all stars of the main sequence of the diagram, is analyzed. The Schönberg-Chandrasekhar limit and its dependence on the nature of stars are analyzed: isothermal, polytropic, etc. The problems of homogeneity and heterogeneity of stars and its influence on the Schönberg-Chandrasekhar limit are observed too. Its role in the development of modern astrophysics is shown. Prospects for the use of the Schoenberg-Chandrasekhar limit for nuclei other than hydrogen and helium are also discussed.

Key words: *thermonuclear processes, Lawson's criterion, Schoenberg-Chandrasekhar limit, deuterium, tritium, modeling.*

Петро ТРОХИМЧУК
кандидат фізико-математичних наук, доцент, доцент кафедри теоретичної та комп'ютерної фізики імені А. В. Свідзинського, Волинський національний університет імені Лесі Українки, просп. Волі, 13, м. Луцьк, Волинська область, Україна, 43025
ORCID ID: *https://orcid.org/0000-0003-2737-0506*
SCOPUS-AUTHOR ID: *8383601100*

Бібліографічний опис статті: Трохимчук, П. (2024). Проблеми моделювання критичних термоядерних процесів. *Фізика та освітні технології*, 2, 71–78, doi: https://doi.org/10.32782/pet-2024-2-10

ПРОБЛЕМИ МОДЕЛЮВАННЯ КРИТИЧНИХ ТЕРМОЯДЕРНИХ ПРОЦЕСІВ

Подано короткий аналіз проблеми моделювання критичних термоядерних процесів. Акцентується увага на двох типах процесів. Перший визначається умовами виникнення та генерації термоядерних реакцій в стаціонарному режимі. Ця проблема є основною для створення термоядерних реакторів і має земне значення. Другий пов'язаний з часом життя стаціонарної фази термоядерного котла. Ця проблема є основною для життя зірок і має вселенське значення. Перша відноситься до проблеми порогу виникнення та генерації термоядерних реакцій і пов'язана з побудовою термоядерних реакторів. Проаналізовано критерій Лоусона та показано його значення в проблемі створення термоядерних реакторів. Виділено та проаналізовано дейтерій-дейтерієву та дейтерій-тритієву реакції. Обговорюються різні механізми моделювання генерації та реалізації цих реакцій, у тому числі за допомогою магнітних полів. Дано основні поняття мюонного каталізу та його роль у виникненні *термоядерних реакцій. Проаналізовано питання про вплив форми та симетрії ядер дейтерію та тритію на поріг генерації термоядерних реакцій і внесок у критерій Лоусона. Друга частина відноситься до астрофізики. Сформульовано критерій Шенберга-Чандрасекара. Проаналізовано теорію Шенберга-Чандрасекара про час перебування зірки на головній послідовності діаграми Герцшпрунга-Ресселя, яка є загальною для всіх зірок головної послідовності діаграми. Проаналізовано межу Шенберга-Чандрасекара та її залежність від природи зірок. Показано розширення теорії Шонберга-Чандрасекара на політропні процеси та неоднорідні системи. Розглянуто проблеми однорідності та неоднорідності зірок та її вплив на межу Шенберга-Чандрасекара. Показано його роль у розвитку сучасної астрофізики. Також обговорюються перспективи використання межі Шенберга-Чандрасекара для ядер, відмінних від водню та гелію.*

Ключові слова: *термоядерні процеси, критерій Лоусона, межа Шенберга-Чандрасекара, дейтерій, тритій, моделювання.*

Introduction. A brief analysis of the problem of modeling critical thermonuclear processes is presented. Attention was focused on two types of processes.

First is determined by the generation of generation of thermonuclear reactions in stationary regime (Abu-Shavareb, 2022; Beringer, 2012; Frank, 1947; Kelly, 2021; Lawson, 1957; Muon, 2024; Petkov, 2012; Tipton, 2015; Trokhimchuck, 2024; Wesson, 2004). This problem is main for the creation thermonuclear reaction and has Earth value. Second is lifetime of stationary phase (Andrievsky, 2007; Chandrasekhar, 1938; Choudhuri, 2023; Schönberg, 1942; Trokhimchuck, 2024). This problem is main for lifetime of stars and have Universe value.

The first refers to the problem of the threshold for the occurrence of thermonuclear reactions. Here, Lawson's criterion is analyzed and its significance in the problem of thermonuclear reactor construction is shown. Deuterium-deuteriun and deuterium-tritium reactions are analyzed. Various mechanisms of modeling the generation and realization of these reactions, including magnetic fields, are discussed. The well-founded concepts of muon catalysis and its role in the generation of thermonuclear reactions are also given. The issue of the influence of the shape and symmetry of deuterium and tritium nuclei on the threshold for the generation of thermonuclear reactions and its contribution to the Lawson criterion is analyzed (Abu-Shavareb, 2022; Lawson, 1942; Petkov, 2012; Tipton, 2015; Trokhimchuck, 2024).

The second part refers to astrophysics. The Schönberg-Chandrasekhar criterion is formulated (Petkow, 2012). The Schönberg-Chandrasekhar theory of the residence time on the main sequence of the Hertzsprung-Ressel diagram, which is general for all stars of the main sequence of the diagram, is analyzed (Schönberg, 1942; Tipton, 2015). The Schönberg-Chandrasekhar limit and its dependence on the nature of stars are analyzed: isothermal, polytropic, etc. (Trokhimchuck, 2024; Trokhimchuck, 2024; Wesson, 2004). The problems of homogeneity and heterogeneity of stars and its influemce on the The Schönberg-Chandrasekhar limit are observed too (Trokhimchuck, 2024; Trokhimchuck, 2024; Wesson, 2004). Its role in the development of modern astrophysics is shown. Prospects for the use of the Schoenberg-Chandrasekhar limit for nuclei other than hydrogen and helium are also discussed.

Lawson's criterion and thermonuclear reactions

Calculations of the power balance in thermonuclear reactors operating under various idealized conditions are given by Lawson (Lawson, 1942; Trokhimchuck, 2024). Two classes of reactors are considered: first, self-sustaining systems in which the charged reaction products are trapped and, secondly, pulsed systems in which the charged reaction products escape so that energy must be supplied continuously during the pulse. It is found that not only must the temperature be sufficiently high, but also the reaction must be sustained long enough for a definite fraction of the fuel to be burnt.

Main thermonuclear reaction are reactions between hydrogen isotopes: deuterium – deuterium, tritium – deuterium. This reactions have little crossectins $\sim 10^{-2}$ barns (Abu-Shavareb, 2022; Lawson, 1942; Petkov, 2012; Tipton, 2015; Trokhimchuck, 2024; Wesson, 2004).

The energy relased per unit time and volume by thermonuclear reactions in a hot gas is given by (Lawson, 1942; Trokhimchuck, 2024-2)

$$P_{reac} = n_1 n_2 v\sigma(T)\, E \,. \tag{1}$$

where n_1 and n_2 are the number densties of the nuclej of thr first and second kinds, and $\langle v\sigma(T)\rangle$ is the product of the relative velocities of the nuclei and the reaction cross-section averaged over the Maxvellian velocity distribution corresponding to the temperature T , and E is the energy released by one reaction.

For D – D reaction this formula may be represented as (Lawson, 1942; Trokhimchuck, 2024-2)

$$P_{reac} = \frac{1}{2} n^2 \langle v\sigma(T) \rangle E \,. \quad (2)$$

Energy can be lost from the hot gas in two ways, by radiation and by conduction. The power radiated per unite voloume in hydrogen is given as (Lawson, 1942; Trokhimchuck, 2024-2)

$$P_B = 1.4 \cdot 10^{-34} n^2 T^{1/2} \text{ watts } \cdot cm^{-3}. \quad (3)$$

Let us give an example of systems in which reaction products are retained. The orders of magnitude involved, the slowing down range of bthe charged reaction products in a gas at 10^8 degrees and 10^4 atmospheres pressure $\left(n = 3 \cdot 10^{17}\, {nuclei}/{cm^3}\right)$ is on the order of kilometre. The range of neutrons is hundreds of kilometres (Lawson, 1942; Trokhimchuck, 2024).

For systems in which the reaction products escape the parameter *R* was introduced; this is ratio of the energy realized in the hot gas to the energy supplied. Now the energy realized by the reraction appears as heat generated in the walls of apparatus, and thus has to be converted to electrical, mechanical or chemical energy before it can be fed back into the gas. If η is the efficiency with which is can be done, then condition for a system with a net power gain is

$$\eta(R+1) > 1. \quad (4)$$

The maximum value of η is about 1/3, so what *R* must be greater than 2 (Lawson, 1942; Trokhimchuck, 2024).

For the our pulsed cycle we have

$$R = \frac{tP_{reac}}{tP_B + 3nkT} = \frac{P_{reac}/3n^2kT}{P_B/3n^2kT + 1/nt}, \quad (5)$$

where P_{reac} and P_B are respectively the reacyion power and radiated p ower per unite volume. The $3nkT$ term represents the energy required to heat the gas to a temperature *T*. Electron binding energies are neglected, but the contribution from electrons is included (this accounts for the factor 3 rather than 3/2) (Lawson, 1942; Trokhimchuck, 2024).

Since P_{reac} and P_B are are both proportional to n^2, R is a function of the T and nt. In Fig. 1 curves R against T for various values of T are shown for D – Dreaction assuming that the tritium formed is also burnt.

Fig. 2 shows similar curvers for T-D reaction (Lawson, 1942; Trokhimchuck, 2024).

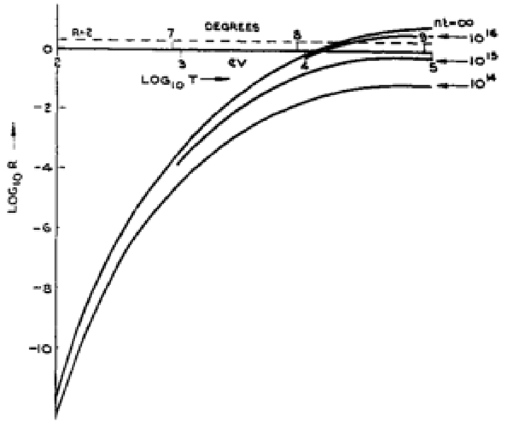

Fig. 1. Variation of *R* with *T* for various values of *nt* for D – D reaction (Lawson, 1942; Trokhimchuck, 2024)

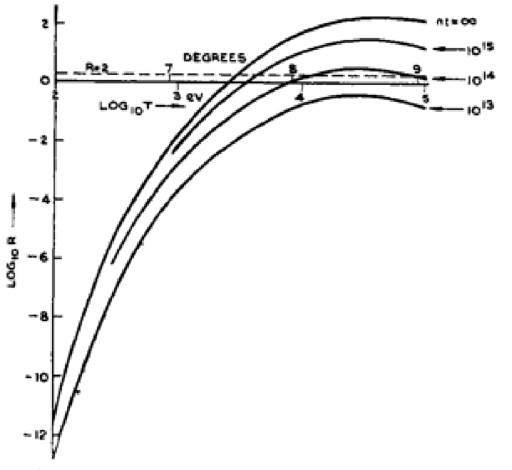

Fig. 2. Variation of *R* with *T* for various values of *nt* for T – D reaction (Lawson, 1942; Trokhimchuck, 2024-2)

By equating radiation losses and the volumetric fusion rates, Lawson estimated the minimum temperature for the fusion for the deuterium–tritium (D-T) reaction (Abu-Shavareb, 2022; Petkov, 2012; Tipton, 2015; Wesson, 2004)

$${}^2_1D + {}^3_1T \to {}^4_2He(3.5MeV) + {}^1_0n(14.1MeV) \quad (6)$$

to be 30 million degrees (2.6 *keV*), and for the deuterium–deuterium (D-D) reaction

$${}^2_1D + {}^2_1De \to {}^3_1T(1.0MeV) + {}^1_1p(3.0MeV) \quad (7)$$

to be 150 million degrees (12.9 *keV*).

The confinement time τ_E measures the rate at which a system loses energy to its environment. The faster the rate of loss of energy, P_{loss} , the shorter the energy confinement time. It is the energy density W (energy content per unit volume) divided by the power loss density P_{loss} (rate of energy loss per unit volume)

$$\tau_E = \frac{W}{P_{loss}}. \quad (8)$$

For a fusion reactor to operate in steady state, the fusion plasma must be maintained at a constant temperature. Thermal energy must therefore be added at the same rate the plasma loses energy in order to maintain the fusion conditions. This energy can be supplied by the fusion reactions themselves, depending on the reaction type, or by supplying additional heating through a variety of methods.

For illustration, the Lawson criterion for the D-T reaction will be derived here, but the same principle can be applied to other fusion fuels. It will also be assumed that all species have the same temperature, that there are no ions present other than fuel ions (no impurities and no helium ash), and that D and T are present in the optimal 50-50 mixture.[a] Ion density then equals electron density and the energy density of both electrons and ions together is given by (Abu-Shavareb, 2022; Petkov, 2012; Tipton, 2015; Wesson, 2004)

$$W = 3nT, \quad (9)$$

where T is the temperature in electronvolt (eV) and n is the particle density.

The volume rate f (reactions per volume per time) of fusion reactions is

$$f = n_D n_T \langle\sigma v\rangle \geq \frac{1}{4} n^2 \langle\sigma v\rangle, \quad (10)$$

where σ is the fusion cross section, v is the relative velocity, and $\langle\ \rangle$ denotes an average over the Maxwellian velocity distribution at the temperature T.

The volume rate of heating by fusion is f times E_{Ch}, the energy of the charged fusion products (the neutrons cannot help to heat the plasma). In the case of the D-T reaction, $E_{ch} = 3.5\ MeV$.

The Lawson criterion requires that fusion heating exceeds the losses (Abu-Shavareb, 2022; Petkov, 2012; Tipton, 2015; Wesson, 2004):

$$fE_{Ch} \geq P_{loss}. \quad (11)$$

Substituting in known quantities yields:

$$\frac{1}{4} n^2 \langle\sigma v\rangle E_{Ch} \geq \frac{2nT}{\tau_E}. \quad (12)$$

Rearranging the equation produces [3-6]:

$$n\tau_E \geq L = \frac{12T}{E_{Ch}\langle\sigma v\rangle}. \quad (13)$$

The quantity $\frac{T}{\langle\sigma v\rangle}$ is a function of temperature with an absolute minimum. Replacing the function with its minimum value provides an absolute lower limit for the product $n\tau_E$. This is the Lawson criterion (Abu-Shavareb, 2022; Petkov, 2012; Tipton, 2015; Wesson, 2004).

For the deuterium–tritium reaction, the physical value is at least

$$n\tau_E \geq 1.5 \cdot 10^{20} \frac{s}{cm^3}. \quad (14)$$

The minimum of the product occurs near $T = 26\ keV$.

The Lawson criterion, or minimum value of (electron density energy confinement time) required for self-heating, for three fusion reactions is bepresented in Fig. 3 (Abu-Shavareb, 2022; Petkov, 2012; Tipton, 2015; Wesson, 2004)

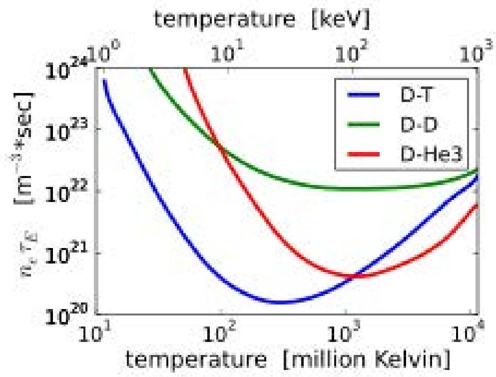

Fig. 3. The Lawson criterion, or minimum value of (electron density energy confinement time) required for self-heating, for three fusion reactions. For DT, $n\tau_E$ minimizes near the temperature 25 keV (300 million kelvins) (Abu-Shavareb, 2022; Petkov, 2012; Tipton, 2015; Wesson, 2004)

A still more useful figure of merit is the "triple product" of density, temperature, and confinement time, $nT\tau_E$. For most confinement concepts, whether inertial, mirror, or toroidal confinement, the density and temperature can be varied over a fairly wide range, but the maximum attainable pressure p is a constant. When such is the case, the fusion power density is proportional to $p^2\langle\sigma v\rangle/T^2$. The maximum fusion power available from a given machine is therefore reached at the temperature T where $\langle\sigma v\rangle/T^2$ is a maximum. By continuation of the above derivation, the following inequality is readily obtained (Abu-Shavareb, 2022; Petkov, 2012; Tipton, 2015; Wesson, 2004):

$$nT\tau_E \geq \frac{12}{E_{ch}}\frac{T}{\langle\sigma v\rangle}. \quad (15)$$

This quantity is also a function of temperature with an absolute minimum at a slightly lower temperature than

For the D-T reaction, the minimum occurs at $T = 14\ keV$. The average <σv> in this temperature region can be approximated as (Abu-Shavareb, 2022; Petkov, 2012; Tipton, 2015; Wesson, 2004)

$$\langle\sigma v\rangle = 1.1\cdot10^{-24}T^2\frac{m^2}{s},\ T \text{ in } keV. \quad (16)$$

So the minimum value of the triple product value at $T = 14\ keV$ is about

$$nT\tau_E \geq 3\cdot10^{21}keV\cdot s/m^3\left(3.5\cdot10^{28}K\cdot s/m^3\right). \quad (17)$$

This number has not yet been achieved in any reactor, although the latest generations of machines have come close. JT-60 reported 1.53x10^{21} $keV{\cdot}s{\cdot}m^{-3}$. For instance, the TFTR has achieved the densities and energy lifetimes needed to achieve Lawson at the temperatures it can create, but it cannot create those temperatures at the same time. ITER aims to do both (Wesson, 2004).

The fusion triple product condition for three fusion reactions are represemted in Fig. 4 (Abu-Shavareb, 2022; Petkov, 2012; Tipton, 2015)

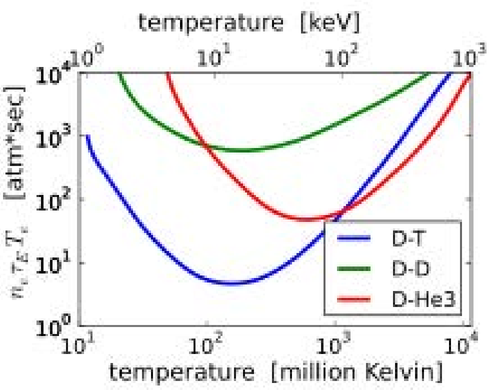

Fig. 4. The fusion triple product condition for three fusion reactions (Abu-Shavareb, 2022; Petkov, 2012; Tipton, 2015)

As for tokamaks, there is a special motivation for using the triple product. Empirically, the energy confinement time τ_E is found to be nearly proportional to $n^{1/3}/P^{2/3}$. In an ignited plasma near the optimum temperature, the heating power P equals fusion power and therefore is proportional to n^2T^2. The triple product scales as

$$nT\tau_E \approx \begin{cases} nT\left(n^{1/3}/P^{2/3}\right); \\ nT\left(n^{1/3}/\left(n^2P^2\right)^{2/3}\right); \\ T^{-1/3}. \end{cases} \quad (18)$$

The triple product is only weakly dependent on temperature as $T^{-1/3}$. This makes the triple product an adequate measure of the efficiency of the confinement scheme (Abu-Shavareb, 2022; Petkov, 2012; Tipton, 2015). The Lawson criterion applies to inertial confinement fusion (ICF) (Tipton, 2015) as well as to magnetic confinement fusion (MCF) (Petkov, 2012) but in the inertial case it is more usefully expressed in a different form. A good approximation for the inertial confinement time is the time that it takes an ion to travel over a distance R at its thermal speed

$$v_{th} = \sqrt{\frac{k_BT}{m_i}}, \quad (19)$$

where m_i denotes mean ionic mass. The inertial confinement time can thus be approximated as

$$\tau_E \approx \frac{R}{v_{th}} = R\sqrt{\frac{m_i}{k_BT}}. \quad (20)$$

By substitution of the above expression into relationship (20), we obtain

$$n\tau_E \approx nR\sqrt{\frac{m_i}{k_BT}} \geq \frac{12}{E_{Ch}}\cdot\frac{k_BT}{\langle\sigma v\rangle}. \quad (21)$$

From where we get the following formula

$$nR \geq \frac{12}{E_{Ch}}\cdot\frac{(k_BT)^{3/2}}{\langle\sigma v\rangle m_i^{1/2}} \quad (22)$$

or

$$nR \geq \frac{(k_BT)^{3/2}}{\langle\sigma v\rangle}. \quad (23)$$

This product must be greater than a value related to the minimum of $T^{3/2}$/<σv>. The same requirement is traditionally expressed in terms of mass density $\rho = \langle nm_i\rangle$:

$$\rho\cdot R \geq 1\,g/cm^2. \quad (24)$$

Satisfaction of this criterion at the density of solid D-T (0.2 g/cm^3) would require a laser pulse of implausibly large energy. Assuming the energy required scales with the mass of the fusion plasma ($E_{laser} \sim \rho R^3 \sim \rho^{-2}$), compressing the fuel to 10^3 or 10^4 times solid density would reduce the energy required by a factor of 10^6 or 10^8, bringing it into a realistic range. With a compression by 10^3, the compressed density will be 200 g/cm^3, and the compressed radius can be as small as 0.05 mm. The radius of the fuel before compression would be 0.5 mm. The initial pellet will be perhaps twice as large since most of the mass will be ablated during the compression (Petkov, 2012; Tipton, 2015).

The fusion power times density is a good figure of merit to determine the optimum temperature for magnetic confinement, but for inertial confinement the fractional burn-up of the fuel is probably more useful. The burn-up should be proportional to the specific reaction rate ($n^2\langle\sigma v\rangle$) times the confinement time (which scales as $T^{-1/2}$) divided by the particle density n (Abu-Shavareb, 2022; Petkov, 2012; Tipton, 2015):

$$\text{burn-up fraction} \approx \begin{cases} n^2 \langle \sigma v\rangle T^{1/2} / n \,; \\ nT \langle \sigma v\rangle / T^{3/2} \,. \end{cases} \quad (25)$$

Thus the optimum temperature for inertial confinement fusion maximises $\langle\sigma v\rangle/T^{3/2}$, which is slightly higher than the optimum temperature for magnetic confinement.

Lawson's analysis is based on the rate of fusion and loss of energy in a thermalized plasma. There is a class of fusion machines that do not use thermalized plasmas but instead directly accelerate individual ions to the required energies. The best-known examples are the migma, fusor and polywell (Abu-Shavareb, 2022; Lawson, 1942; Petkov, 2012; Tipton, 2015; Trokhimchuck, 2024; Wesson, 2004).

When applied to the fusor, Lawson's analysis is used as an argument that conduction and radiation losses are the key impediments to reaching net power. Fusors use a voltage drop to accelerate and collide ions, resulting in fusion. The voltage drop is generated by wire cages, and these cages conduct away particles (Abu-Shavareb, 2022; Lawson, 1942; Petkov, 2012; Tipton, 2015; Trokhimchuck, 2024; Wesson, 2004).

Polywells are improvements on this design, designed to reduce conduction losses by removing the wire cages which cause them. Regardless, it is argued that radiation is still a major impediment.

Muon-catalyzed fusion (abbreviated as μCF or MCF) is a process allowing nuclear fusion to take place at temperatures significantly lower than the temperatures required for thermonuclear fusion, even at room temperature or lower (Beringer, 2012; Frank, 1947; Kelly, 2021; Muom, 2024). It is one of the few known ways of catalyzing nuclear fusion reactions.

Muons are unstable subatomic particles which are similar to electrons but 207 times more massive. If a muon replaces one of the electrons in a hydrogen molecule, the nuclei are consequently drawn 186 times closer than in a normal molecule, due to the reduced mass being 186 times the mass of an electron. When the nuclei move closer together, the fusion probability increases, to the point where a significant number of fusion events can happen at room temperature (Beringer, 2012; Frank, 1947; Kelly, 2021; Muom, 2024).

Methods for obtaining muons, however, require far more energy than can be produced by the resulting fusion reactions. Muons have a mean lifetime of 2.2 μs (Beringer, 2012), much longer than many other subatomic particles but nevertheless far too brief to allow their useful storage.

To create useful room-temperature muon-catalyzed fusion, reactors would need a cheap, efficient muon source and/or a way for each individual muon to catalyze many more fusion reactions (Beringer, 2012; Frank, 1947; Kelly, 2021; Petkov, 2012).

From our point of view, the following additional studies should be conducted for optimal modeling of efficient thermonuclear reactors: 1. Choose the conditions of the experiment so that the majority of mesonuclei participate in synthesis reactions. 2. To take into account the geometric intersection of synthesis reactions and to select appropriate nuclei and the geometry of the experiment for this purpose. 3. More widely apply impulse processes for initial detonation and obtaining starting conditions for obtaining the required reaction characteristics.

The Schönberg-Chandrasekhar limit and astrophysics. In stellar astrophysics, the Schönberg–Chandrasekhar limit (Chandrasekhar, 1938; Schönberg, 1942; Trokhimchuck, 2024-1) is the maximum mass of a non-fusing, isothermal core that can support an enclosing envelope. It is expressed as the ratio of the core mass to the total mass of the core and envelope. Estimates of the limit depend on the models used and the assumed chemical compositions of the core and envelope; typical values given are from 0.10 to 0.15 (10% to 15% of the total stellar mass). This is the maximum to which a helium-filled core can grow, and if this limit is exceeded, as can only happen in massive stars, the core collapses, releasing energy that causes the outer layers of the star to expand to become a red giant. It is named after the astrophysicists Subrahmanyan Chandrasekhar and Mario Schönberg, who estimated its value in a 1942 paper (Schönberg, 1942). They estimated it to be

$$\left(\frac{M_c}{M}\right) = 0.37\left(\frac{\mu_e^2}{\mu_c^2}\right), \quad (26)$$

where M is the mass, μ is the mean molecular weight, index c denotes the core, and index e is the envelope.

The Schönberg-Chandrasekhar limit comes into play when fusion in a main-sequence star exhausts the hydrogen at the center of the star. The star then contracts until hydrogen fuses in a shell surrounding a helium-rich core, both of which are surrounded by an envelope consisting primarily of hydrogen. The core increases in mass as the shell burns its way outwards through the star. If the star's mass is less than approximately 1.5 solar masses, the core will become degenerate before the Schönberg-Chandrasekhar limit is reached, and, on the other hand, if the mass is greater than approximately 6 solar masses, the star leaves the main sequence with a core mass already greater than the Schönberg-Chandrasekhar limit so its core is never isothermal before helium fusion. In the remaining case, where the mass is between 1.5 and 6 solar masses, the core will grow until the limit is reached, at which point it will contract rapidly until helium starts to fuse in the core.

In astrophysics, as a rule, stationary processes take place. This is especially true for stars that are on the main sequence of the Hertzsprung-Ressel diagram (Andrievsky, 2007; Chandrasekhar, 1938; Schönberg, 1942; Trokhimchuck, 2024-1). The life time of the stars on this diagram, depending on their spectral class, lasts from several million years to 100 million years. The stay of the star on the main sequence lasts until its nuclear fuel – hydrogen – is exhausted in its superstructure. More precisely, until, as established by M. Schönberg and S. Chandrasekhar, a helium nucleus with a mass of 10-20 percent of the mass of the Sun is formed in the center of the star.

The time it takes for a star to reach the Schönberg-Chandrasekhar evolutionary limit (that is, the time it spends on the leading sequence of the Hertzsprung-Ressel diagram) is estimated by the formula (Chandrasekhar, 1938; Trokhimchuck, 2024):

$$t_{LS} \sim \frac{M}{L} \cong 10^{10}\left(\frac{M_G}{M}\right)^{-2.5} \textit{years}.$$

where M is the mass of the star in the masses of the Sun M_G, L – the luminosity of the star in the luminosities of the Sun. Here it is taken into account that the luminosity of the star is $L \sim M^{-3.5}$ (Chandrasekhar, 1938; Trokhimchuck, 2024-1) and that the reserves of thermonuclear energy are proportional to the total mass of the star. The final stage of this evolution is the formation of a red giant or supergiant (Chandrasekhar, 1938; Trokhimchuck, 2024-1).

The existence of a maximum isothermal core mass fraction (q_{max}), the Schönberg-Chandrasekhar limit, is one of the 'classic' results from the theory of stellar structure. This limit can be demonstrated through a simplified composite polytrope model in which an isothermal core is surrounded by an $n = 1$ polytrope envelope. While this model underestimates q_{max} by 25 % in the homogeneous case, it is accurate to within 5 % in the more realistic inhomogeneous situation (Beech, 1988).

The Schönberg-Chandrasekhar limit in post-main-sequence evolution for stars os masses vin the range $1.4 \le M / M_G \le 6.0$ gives the maximum pressure that the stellar core can withstand, omce of the central hydrogen is exhausted (Choudhuri, 2023). It is usually expressed as a quadratic function of $1/\alpha$, with α being the ratio of the mean molecular weight of the core to that of the envelope. Here, we revisit this vlimit in scenarios where the pressure balsance equation in the stellar interior b may be modified, amd in the presemce of small stellar pressure anisotropy, the might arise due to several physical phenomena. Using numerical analysis, we derive a three parameter-dependent master formula for the limit, and discuss various physical consequences. As a by-product, in a limiting case of our formula, wr find that in the standard Newtonian framework, the Schönberg-Chandrasekhar limit is best-fit by a polynomial that is linear. Rather than quadratic, to lowest order in $1/\alpha$ (Choudhuri, 2023).

From our point of view, the Schönberg-Chandrasekhar theory should be extended to heavier chemical elements and to more short-lived and long-lived processes.

Conclusions. The problem of generation the stationary thermonucleas reactions is discussed.

Lawson's criterion and its application for the estimation the critical regimes of thermonuclear reactions are analyzed.

Ways of develop the more widely applications of Lawson's criterion are discussed.

Main peculiarities of Schönberg-Chandrasekhar limit in astrophysics and expansion area of its applications are observed.

REFERENCES:

1. Abu-Shawareb, H. et al. (3.5 pages) (2022). Lawson Criterion for Ignition Exceeded in an Inertial Fusion Experiment. *Physical Review Letters,* 129, 075001, 17 p.

2. Beringer, J. et al. (0.5 page) (2012). Review of Particle Physics. Phys. Rev. D 86, 010001, 1526 p.

3. Frank, F.C. (1947). Hypothetical Alternative Energy Sources for the 'Second Meson' Events. Nature, vol. 160(4068), pp. 525–527.

4. Kelly, R. S., Hart, L. J. F., & Rose, S. J. (2021). An investigation of efficient muon production for use in muon catalyzed fusion". Journal of Physics: Energy, vol. 3 (3), pp. 525–527.

5. Lawson, J. D. (1957). Some Criteria for the Power Producing Thermonuclear Reactor. Proc. *Of the Physical Society*, Sec. B, 70(1), pp. 6-10.

6. Muon-catalyzed fusion (2024). URL: https://en.wikipedia.org/wiki/Muon-catalyzed_fusion

7. Petkow, D., Gabrielli, R.A., Herdrich, G., R. Laufer, R., & Röse, H.-P. (2012). Generalized Lawson criterion for magnetic fusion applications in space. *Fusion Engineering and Design*, Vol. 87, is.1, pp. 30–38.

8. Tipton, R. E. (2015). *Generalized Lawson Criteria for Inertial Confinement Fusion*. Lauremce Livermore National Laboratory. Technical Report LLNL-TR-676592, 2015, 59 p.

9. Trokhimchuck, P. P. (2024). To Question about Main Principles and Criteria of Critical Processes and Phenomena, IJARPS, vol. 11, is. 6, pp. 1–17.

10. Wesson, J. (2004). Tokamaks. Oxford: Claremdon Press, 750 p.

11. Andrievsky, S. M., & Klymyshyn, I. A. (2007). Kurs zagalnoyi astronomiyi [General astronomy course]. Odessa: Astroprint, 476 p. [in Ukrainian].

12. Chandrasekhar, S. (1938). Introduction to the Study of Stellar Structure. Chicago: University Press, 508 p.

13. Choudhuri, S., & Tapobrata, S. (2023). The Schönberg-Chandrasekhar limit ln presence of small anisotropy and modified gravity. Monthly Notices of the Royal Astronomical Society, vol. 523, 2023, pp. 518–526.

14. Schönberg, M., & Chandrasekhar, S. (1942). On the evolution of main-sequence star. *The Astrophysical Journal*, Vol. 96, Is.2, pp.161–172.

15. Trokhimchuck, P. P. (2024). Some Problems of Modeling Astrophysical and Cosmological Critical Processes. In: Eds. Jayminkumar Rajanikant Ray @ Shyam Sunder Sharma. Recent Review and Research in Physics, Vol. 10, Ch. 5. New Dehli: AkiNik Publications, pp. 65-94.

16. Beech, M. (1988). The Schönberg-Chandrasekhar limit: Polytropic Approximation. *Astrophysics and Space Science*, vol. 147, pp. 219–227.

ЛІТЕРАТУРА:

1. Abu-Shawareb H. et al. (3.5 pages) Lawson Criterion for Ignition Exceeded in an Inertial Fusion Experiment. *Physical Review Letters.* 2022. 129. 075001. 17p.

2. Beringer J. et al. (0.5 page). Review of Particle Physics. Phys. Rev. 2012. D 86, 010001, 1526 p.

3. Frank F.C. Hypothetical Alternative Energy Sources for the 'Second Meson' Events. Nature. 1947. vol. 160(4068). pp. 525–527.

4. Kelly R. S., Hart L. J. F., Rose S. J. An investigation of efficient muon production for use in muon catalyzed fusion". Journal of Physics: Energy, 2021.vol. 3 (3), pp. 525–527.

5. Lawson J. D. Some Criteria for the Power Producing Thermonuclear Reactor. Proc. *Of the Physical Society*, Sec. B, 1957, 70(1), pp. 6–10.

6. Muon-catalyzed fusion. 2024. URL: https://en.wikipedia.org/wiki/Muon-catalyzed_fusion

7. Petkow D., Gabrielli R.A., Herdrich G., R. Laufer R., Röse H.-P. Generalized Lawson criterion for magnetic fusion applications in space. *Fusion Engineering and Design*, 2012, Vol. 87, is.1, pp.30-38.

8. Tipton R. E. *Generalized Lawson Criteria for Inertial Confinement Fusion*. Lauremce Livermore National Laboratory. Technical Report LLNL-TR-676592, 2015, 59 p.

9. Trokhimchuck P. P. To Question about Main Principles and Criteria of Critical Processes and Phenomena, IJARPS, 2024, vol. 11, is. 6, pp. 1-17.

10. Wesson J. *Tokamaks*. Oxford: Claremdon Press, 2004, 750 p.

11. Андрієвський С. М., Климишин І. А. Курс загальної астрономії. Одеса: Астропринт, 476 с.

12. Chandrasekhar S. Introduction to the Study of Stellar Structure. Chicago: University Press, 1938, 508 p.

13. Choudhuri S., Tapobrata S. The Schönberg-Chandrasekhar limit ln presence of small anisotropy and modified gravity. *Monthly Notices of the Royal Astronomical Society*, vol. 523, 2023, pp. 518–526.

14. Schönberg M., Chandrasekhar S. On the evolution of main-sequence star. *The Astrophysical Journal*, 1942, Vol. 96, Is.2, pp. 161–172.

15. Trokhimchuck P. P. Some Problems of Modeling Astrophysical and Cosmological Critical Processes. In: Eds. Jayminkumar Rajanikant Ray @ Shyam Sunder Sharma. *Recent Review and Research in Physics*, 2024, Vol. 10, Ch. 5. New Dehli: AkiNik Publications, pp. 65–94.

16. Beech M. The Schönberg-Chandrasekhar limit: Polytropic Approximation. Astrophysics and Space Science, 1988, vol. 147, pp. 219–227.